Comparison of Wavelet- and Time-Marching-Based Microwave Circuit Transient Analyses

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Abstract— **In this paper we derive a transient analysis formulation that can be used in conjunction with wavelets or time-marching methods. The number of unknowns in the formulation is proportional to the number of state variables of the nonlinear devices in the circuit. The formulation was implemented in a general circuit simulator. We evaluate the numerical performance of transient analysis using wavelets and the backward Euler method by simulating a nonlinear transmission line and a quasi-optical grid amplifier. The quasi-optical example illustrates the integration of full-wave electromagnetic analysis in transient circuit simulation.**

I. Introduction

The essence of transient simulation of circuits is solving a system of coupled algebraic and ordinary differential equations. Circuit simulators convert this system into a nonlinear algebraic system of equations. In conventional simulation techniques the number of nonlinear unknowns is approximately equal to the number of nodes in the circuit. Circuit simulation methods based on the state variables of the nonlinear devices allow the analysis of circuits with the minimum number of unknowns and error functions. This approach has several advantages. In microwave circuits, the resulting system of nonlinear equations is generally much smaller than the nonlinear system resulting from applying conventional formulations. Another advantage is the robustness and flexibility provided by the state-variable approach [1].

Multiresolution analysis has been used with a wide variety of modeling problems including signal processing and electromagnetics. Zhou *et al.* presented a pseudo-wavelet collocation method applied to the transient simulation of circuits [2, 3]. In Reference [4] we used this collocation method to derive the state-variable-based transient analysis. The analysis is implemented in our circuit simulator, *Transim* [5].

This work was supported by the Multidisciplinary University Research Initiative through the Army Research Office, Grant No. DAAG55-97-K-0132 and by Cadence Design Systems.

We first review and expand the formulation of the transient analysis of Reference [4] in Section II. Although our formulation was originally developed to be used with wavelet transformations, it can also be used with implicit time-marching methods. As an example, we derive a state-variable-based transient analysis using the Backward Euler (BE) formula by just replacing two matrices in the wavelet formulation. In Section III the numerical performance of the transient analysis using wavelets and the BE method is evaluated by simulating a nonlinear transmission line and a quasi-optical grid amplifier.

II. Formulation of the Transient Analysis

Given a function $g(t)$ defined in an interval $I = [0, L]$, where L is an integer number, the following square matrices $\mathbf{W}$ and $\mathbf{W}'$ are defined [4, 6]:

$$\mathbf{g} = \mathbf{W}\hat{\mathbf{g}}_J, \quad \mathbf{g}' = \mathbf{W}'\hat{\mathbf{g}}_J, \tag{1}$$

where $\mathbf{g}$, $\mathbf{g}'$ are vectors whose elements are the function and derivatives values, respectively, at the collocation points and $\hat{\mathbf{g}}_J$ is the vector of the corresponding coefficients. J is the maximum subspace level being considered. Fig. 1 shows the nonzero structure of $\mathbf{W}$ and $\mathbf{W}'$ for the particular wavelet basis functions used in this work.

The final linear circuit equation, from [4], is

$$\mathbf{M}_J\hat{\mathbf{u}}_J = \mathbf{s}_{f,J} + \mathbf{T}_{1,J}\mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J), \tag{2}$$

where $\hat{\mathbf{u}}_J$ is the vector of wavelet coefficients of the nodal voltages and selected currents, $\hat{\mathbf{x}}_J$ is the vector with wavelet coefficients of the state variables of the nonlinear devices, $\mathbf{s}_{f,J}$ is the vector of independent sources, the matrix $\mathbf{T}_{1,J}$ is defined in [4] and $\mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J)$

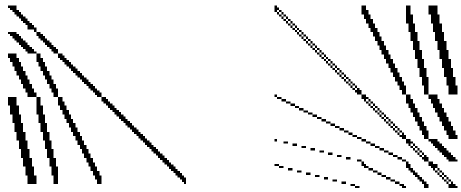

Fig. 1. Representation of the nonzero elements of the transformation matrices for $L = 10$ and $J = 2$: (a) $\mathbf{W}$ and (b) $\mathbf{W}'$.

is the vector of currents at the nonlinear device ports. The error function $\mathbf{F}(\hat{\mathbf{x}}_J)$ is defined as

$$\mathbf{F}(\hat{\mathbf{x}}_J) = \mathbf{T}_{2,J}\hat{\mathbf{u}}_J - \mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J) = 0,$$

where $\mathbf{T}_{2,J}$ is a matrix defined in [4] and $\mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J)$ is the vector of voltages at the nonlinear device ports. Combining the preceding with (2),

$$\begin{aligned}\mathbf{F}(\hat{\mathbf{x}}_J) &= \mathbf{T}_{2,J}\mathbf{M}_J^{-1}\mathbf{s}_{f,J}\\ &\quad+\mathbf{T}_{2,J}\mathbf{M}_J^{-1}\mathbf{T}_{1,J}\mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J) - \mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J).\end{aligned}$$

By defining the compressed source vector $\mathbf{s}_{sv,J} = \mathbf{T}_{2,J}\mathbf{M}_J^{-1}\mathbf{s}_{f,J}$ and the compressed impedance matrix $\mathbf{M}_{sv,J} = \mathbf{T}_{2,J}\mathbf{M}_J^{-1}\mathbf{T}_{1,J}$, we can express the error function as

$$\mathbf{F}(\hat{\mathbf{x}}_J) = \mathbf{s}_{sv,J} + \mathbf{M}_{sv,J}\mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J) - \mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J) = 0. \quad (3)$$

The initial conditions of the entire linear subcircuit are embedded in $\mathbf{s}_{sv,J}$. The system of nonlinear algebraic equations (3) is solved using a globally convergent quasi-Newton method. The size of $\mathbf{M}_{sv,J}$ is $(m-1)n_s \times (m-1)n_s$, where m is the number of collocation points. If the time interval to be simulated requires many collocation points, the nonlinear system to be solved becomes very large. One way to overcome this problem is to divide the total simulation time interval into smaller *windows*. Then solve one time window at a time. The final time sample at each window becomes the initial condition for the next and the method is applied for all windows.

A. Initial Conditions in the State Variables

The wavelet coefficients of each state variable are not completely independent. There is a constraint imposed by the initial condition of the transient analysis. Therefore, the first transform coefficient is excluded from the unknowns. Given the initial condition x_0 and the remaining coefficients $\hat{\mathbf{x}}$, it is possible to obtain the vector of the remaining time samples $\mathbf{x}$ as follows

$$\mathbf{x} = (\mathbf{W}_r - \frac{\mathbf{w}_{c0}}{w_{0,0}}\mathbf{w}_{r0})\hat{\mathbf{x}} + \frac{\mathbf{w}_{c0}}{w_{0,0}}x_0,$$

where $\mathbf{W}_r$ is equal to $\mathbf{W}$ reduced by the first row and column, $\mathbf{w}_{c0}$ and $\mathbf{w}_{r0}$ are the first column and row of $\mathbf{W}$, respectively, excluding the first element $w_{0,0}$.

A similar expression can be obtained for $\mathbf{x}'$, namely

$$\mathbf{x}' = (\mathbf{W}'_r - \frac{\mathbf{w}'_{c0}}{w_{0,0}}\mathbf{w}_{r0})\hat{\mathbf{x}} + \frac{\mathbf{w}'_{c0}}{w_{0,0}}x_0.$$

Higher order derivatives were not used in the present work.

B. Alternative Formulations

An alternative to the transient formulation is to express the nonlinear error function in terms of the wavelet coefficients of the port voltages. This yields a similar error function where $\mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J)$ must be transformed from the physical to the coefficient space (using $\mathbf{W}^{-1}$). At first, this approach would seem to be less efficient because it requires the implementation of inverse wavelet transformation. Nevertheless, this approach would allow the reduction of the linear system size if some coefficients of the port voltages are known to be zero. This can not be done in the physical space.

Other types of analysis can be derived by modifying our transient formulation. For example, by replacing the equations relating the transient initial conditions with certain boundary conditions we can obtain a formulation for the periodic steady-state of a circuit. These alternatives will not be developed here.

C. Transient Using the Backward Euler Formula

The formulation presented in this paper can be applied not only with wavelet transformations but also with other transformations as well. In particular, some implicit time marching methods can be implemented. If the transformation matrices in (1) are defined as follows

$$\mathbf{W} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \text{ and } \mathbf{W}' = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad (4)$$

then the preceding derivation becomes a formulation for a state-variable-based transient analysis using the BE formula. In this case the linear system to be solved is twice as large as the original MNAM (because the transformations matrices are 2×2) and the size of the nonlinear system resulting from (3) is equal to the number of state variables. Multi-step time-marching schemes can also be implemented by choosing adequate $\mathbf{W}$ and $\mathbf{W}'$ and introducing some minor variations in the formulation.

III. Simulation Results and Discussion

In the following we compare the performance of the transient analysis in Transim using wavelets and the BE formula. Wavelet transient analysis was implemented first. To implement the BE transient analysis, we modified $\mathbf{W}$ and $\mathbf{W}'$ in the wavelet transient analysis in Transim to use the definition in (4). The rest of the program was unchanged. The purpose of this was to demonstrate the generality of the formulation and to compare the two transient analysis methods using almost exactly the same code. Note however that in general it is not practical to use (3) with time-marching methods because formulations that are simpler and more efficient exist as the authors show in Chapter 6 of Reference [6].

A. Nonlinear Transmission Line

The wavelet transient analysis in Transim was validated with the comparison of the simulation results of a 47-section nonlinear transmission line with the results of a Spice simulation [4]. The same wavelet transient simulation is compared here with a BE transient simulation.

Fig. 2 shows the simulated voltage of the diode near the load using wavelet transient and backward Euler. Table I compares key simulation parameters. The BE

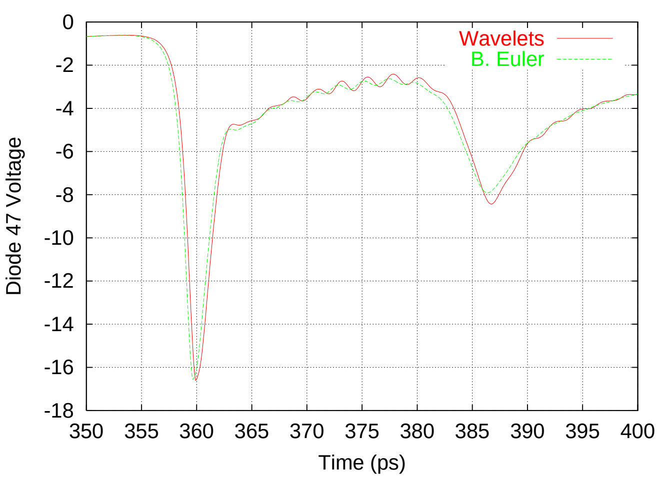

Fig. 2. Comparison of the voltage of the diode close to the load (diode 47) of the nonlinear transmission line.

method is known to introduce numerical damping [7]. Despite the large number of time samples used in BE transient to minimize this effect, the numerical damping is still noticeable in Fig. 2. This does not occur when using trapezoidal integration [4,6]. Although this

TABLE I

Comparison of Spice and wavelet transient simulations of the first 400 ps transient response of the soliton line.

	Wavelets	B. Euler
Time (minutes)	50	17
Memory (MB)	41	56
Scalar Unknowns	564	47
Windows / Time Samples	94	40000

implementation of a time marching transient analysis is inefficient, it is several times faster than the simulation using wavelets. This is because the solution of the nonlinear system in wavelet transient presents two problems: the initial guess in the Newton method is not close to the solution and the number of nonlinear unknowns grows quickly with the window resolution despite the state-variable reduction. If the circuit being simulated has a periodic excitation, the first problem can be addressed by choosing the time window size to be equal to the period. Then the solution for a given time window can be used as a good guess for the solution at the next window. Unfortunately, for this circuit this approach implies a time window too large to be to be handled efficiently. The second problem can be alleviated by using an adaptive scheme to eliminate the coefficients that are known to be zero from the list of unknowns.

B. Grid Amplifier

We evaluate the turn-on transient of a quasi-optical grid amplifier system [8]. The grid structure was modeled using a MOM field simulator [9] to generate the multi-port admittance matrix and excitation currents for the grid structure. To use this data in a time domain analysis such as wavelet or BE transient, the frequency-defined parameters are first approximated by a rational transfer function [6].

The transient simulation of the grid amplifier using convolution was presented in Reference [10]. The initial transient is given in Fig. 3. The microwave excitation is applied at $t = 2$ ns. Note the two different convolution results. If the convolution analysis is performed using the pole-zero model of the grid, the agreement with the other transient simulations is much better. This sug-

TABLE II
Comparison times of the different simulation methods

Description	Convolution (hh:mm:ss)	Wavelets (hh:mm:ss)	Backward Euler (hh:mm:ss)
Bias-on (12 μs)	16:15:00	00:00:37	00:00:48
Bias + Excitation (4 μs)	-	18:24:00	05:16:00
Bias + Excitation (4 ns)	00:09:34	00:04:40	00:00:18

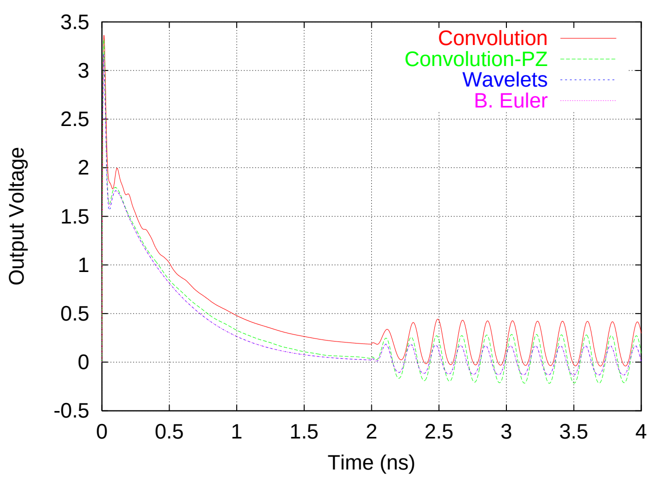

Fig. 3. Transient response at one of the MMIC output

gests that the pole-zero model should be improved. A comparison of the run times for this and other simulations of the grid amplifier is given in Table II. Convolution transient is always the slowest method. Nevertheless, this method can potentially achieve the best accuracy. BE transient is generally the fastest method. However, sometimes transient analysis using wavelets can be faster as shown in the Bias-on simulation in Table II. For very long transient simulations, the only viable alternative to perform the transient analysis seems to be the approximation of the grid network parameters using rational functions. It should be noted that this example includes the incorporation of a full-wave electromagnetic analysis into a transient simulator. Of significance is that the electromagnetic environment is critical to operation and does not represent a circuit parasitic.

IV. Conclusions

Transim is, to the knowledge of the authors, the most advanced wavelet-based nonlinear circuit simulator developed to date[1]. The simulation examples presented here are the most complex circuits ever simulated using wavelets. The simulation results show that time marching transient is faster than wavelet transient with fixed resolution. More research is required before circuit simulation techniques using wavelets can be more efficient than time-marching techniques. In particular the implementation of dynamic variation of resolution including variable resolution at different circuit nodes is required.

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